Dutch Day of Combinatorics 2022

Eindhoven University of Technology, The Netherlands
12 May 2022

https://doc2022.weebly.com/

We welcome you to the Dutch Day of Combinatorics 2022. This one day hybrid event is intended to share recent research activities with one another, and thus strengthen and further build the combinatorics activity in the Netherlands. The goal is for this to be an annual event hosted at different universities in the Netherlands every year, with TU/e hosting the first event.

Programme

09:30	Registration		
09:50	Welcome		
		Speaker	**Title**
	Session I		
10:00		Mariot	Enumerating Coprime Polynomials over $GF(2)$ with Nonzero Constant Term
10:20		Wötzel	On a nonabelian Kneser theorem
10:40		Cames van Batenburg	Short transformations between list colourings
	Plenary talk I		
11:00		Carla Groenland	Graph reconstruction from partial information
11:45	Coffee break		
	Session II		
12:00		de Boer	Uniqueness of the Gibbs measure for the anti-ferromagnetic Potts model on the infinite Δ-regular tree for large Δ
12:20		Huijben	Chromatic zeros of series-parallel graphs
12:40		Bencs	On the number of forests and trees in large regular graphs
13:00	Group photo		
13:10	Lunch		(at the Eurandom lounge, next to MF11/12)
	Session III		
14:00		Polak	New lower bounds on crossing numbers of $K_{m,n}$ from permutation modules and semidefinite programming
14:20		Verstraaten	Cycles in Mallows random permutations
	Plenary talk II		
14:40		Ross Kang	From local sparsity to global
15:25	Coffee break		
	Session IV		
15:40		Denaux	Higgledy-piggledy sets in projective spaces
16:00		Cambie	Extremal Entropy of Graphs
16:20		Dong	The minimum and maximum graphical function-index tree problems
16:40		Wu	Anti-Ramsey numbers for disjoint triangles
17:00	Closing		
17:15	Drinks and borrel		(at Zwarte Doos)

Registered participants

Aida Abiad, Eindhoven University of Techology & Ugent & VUB
Mehmet Akif Yildiz, UvA
Bram Bekker, TU Delft
Ferenc Bencs, UvA
Altan Berdan Kilic, Eindhoven University of Technology
Anurag Bishnoi, TU Delft
Danny Blom, Eindhoven University of Technology
Sander Borst, CWI
Jop Briët, CWI
Daniel Brosch, CWI
Pjotr Buys, University of Amsterdam
Stijn Cambie, IBS
Wouter Cames van Batenburg, TU Delft
Yixin Cao, Hong Kong Polytechnic University
Ke Chen, Penn state university
Jozefien D'haeseleer, Ghent University
Daniel Dadush, CWI
Shagnik Das, National Taiwan University
Maarten De Boeck, University of Rijeka
David de Boer, KdVI
Bart De Bruyn, Ghent University
Jari de Kroon, Eindhoven University of Technology
Lins Denaux, Ghent University
Yanni Dong, University of Twente
Huib Donkers, Eindhoven University of Technology
Joanna Ellis-Monaghan, University of Amsterdam
Robbert Fokkink, TU Delft
Estéban Gabory, CWI
Dion Gijswijt, TU Delft
Carla Groenland, Utrecht
Harald Gropp, Heidelberg
Anina Gruicia, Eindhoven University of Technology
Willem Haemers, Tilburg University
Christopher Hojny, Eindhoven University of Technology
Jeroen Huijben, UvA
Cor Hurkens, Eindhoven University of Technology
Hany Ibrahim, Hochshule Mittweida
Bart Jansen, Eindhoven University of Technology
Shivesh K. Roy, Eindhoven University of Technology
Eda Kaja, TU Darmstadt
Ross Kang, Radboud University
Danish Kashaev, CWI
Eng Keat Hng, Czech technical university of Prague
Judith Keijsper, Eindhoven University of Technology
Steven Kelk, Maastricht University
Antonina Khramova, Eindhoven University of Technology
Pieter Kleer, Tilburg University
Julia Komjathy, TU Delft
Lyuben Lichev, University Jean Monnet, Saint Etienne
Mairon Mahzoun, Eindhoven University of Technology

Isja Mannens, Utrecht
Prabhu Manyem, Nanchang Institute of Technology
Luca Mariot, Radboud University
Lucas Meijer, Utrecht
Ruben Meuwese, Maastricht University
Tobias Muller, Rijksuniversiteit Groningen
Bahar Okumusoglu, Sabanci University
Fernando Oliveira, TU Delft
Lale Ozkahya, Hacettepe University
Antonina P. Khramova, Eindhoven University of Technology
Alex Pellegrini, Eindhoven University of Technology
Rudi Pendavingh, Eindhoven University of Technology
Sven Polak, CWI
Gleb Polevoy, Paderborn University
Ali Rajaei, TU Delft
Guus Regts, University of Amsterdam
Nick Reniers, Eindhoven University of Technology
Arman Rouhani, Maastricht University
Ashkan Safari, Maastricht University
Vibha Sahlot, Charles university
Vincent Schmeits, UvA
Yilun Shang, Northumbria University
Robin Simoens, Ghent University
Lucas Slot, CWI
Bart Smeulders, Eindhoven University of Technology
Hossein Teimoori, Allameh Tabataba'i University of Tehran Iran
Leonidas Theocharous, Eindhoven University of Technology
Edwin van Dam, Tilburg University
Jasper van Doornmalen, Eindhoven University of Technology
Josse van Dobben de Bruyn, TU Delft
Marten van Dijk, CWI
Lucy Verberk, Eindhoven University of Technology
Teun Verstraaten, Rijksuniversiteit Groningen
Jan Volek, Czech technical university of Prague
Matthias Walter, University of Twente
Maximilian Wötzel, Radboud University
Fangfang wu, University of Twente
Xinyi Xu, LSE
Jonathan Zandee, TU Delft
Sjanne Zeijlemaker, Eindhoven University of Technology
Alberto Ravagnani, Eindhoven University of Technology

Scientific organising committee

Aida Abiad (TU/e, UGhent, VUB), Anurag Bishnoi (TUDelft), Jo Ellis Monaghan (UvA), Frits Spieksma (TU/e)

Local organisers

Wout Moutmaker, Lucy Verberk, Antonina Khramova, Anita Klooster

Sponsorship

The meeting was made possible through the support of Networks and Eindhoven University of Technology.

Abstracts of the plenary talks

Title: From local sparsity to global

Presenter: Ross Kang
Affiliation: Radboud University, The Netherlands
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I will survey a number of well-known Ramsey-type problems in graph theory, mostly related to finding good colourings or large independent sets. These problems all have in common the idea that to obtain good global sparse graph structure, one can often leverage conditions on local structure. I'll attempt to give an intuitive overview of two main tools for such problems, both of which link to fundamental aspects of the Lovasz local lemma, and its connections to other areas such as randomised algorithms and statistical physics. Time permitting, I can highlight some key outstanding challenges.

The talk will touch on various joint works with Alon, Cambie, Davies, Hurley, de Joannis de Verclos, Pirot, Sereni.

Title: Graph reconstruction from partial information

Presenter: Carla Groenland
Affiliation: Utrecht University, The Netherlands
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The graph reconstruction conjecture states that each graph G on at least 3 vertices can be reconstructed from the multiset of all induced subgraphs on n-1 vertices. Although this conjecture is notoriously difficult, it is easy to reconstruct some information about the graph, e.g. the degree sequence of G and whether G is connected. Moreover, some graphs with extra structure can be reconstructed, such as trees. We study a set-up with smaller induced subgraphs ('small cards'). We use a surprising algebraic tool and introduce several counting techniques.

This is based on joint work with Tom Johnston, Alex Scott and Jane Tan.

Abstracts of the contributed talks

Title: On the number of forests and trees in large regular graphs

Presenter: Ferenc Bencs
Affiliation: Korteweg de-Vries Institute, UvA
E-mail: f.bencs@uva.nl

For a graph $G = (V, E)$ let $T_G(x, y)$ be the Tutte-polynomial. Let $(G_n)_n$ be a sequence of d-regular graphs with girth $g(G_n) \to \infty$, the length of the shortest cycle, then the limit

$$\lim_{n\to\infty} T_{G_n}(x,y)^{1/v(G_n)} = \begin{cases} (d-1)\left(\frac{(d-1)^2}{(d-1)^2-x}\right)^{d/2-1} & \text{if } x \leq d-1, \\ x\left(1+\frac{1}{x-1}\right)^{d/2-1} & \text{if } x > d-1. \end{cases}$$

for $x \geq 1$ and $0 \leq y \leq 1$. If $(G_n)_n$ is a sequence of random d-regular graphs, then the statement holds true asymptotically almost surely.

This theorem generalizes results of McKay ($x = 1, y = 1$, spanning trees of random d-regular graphs) and Lyons ($x = 1, y = 1$, spanning trees of large-girth d-regular graphs). Interesting special cases are $T_G(2, 1)$ counting the number of spanning forests, $T_G(2, 0)$ counting the number of acyclic orientations.

This is joint work with Péter Csikvári.

Title: Extremal entropy of graphs

Presenter: Stijn Cambie
Affiliation: Radboud, Warwick, IBS
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Entropy is a concept and physical property that is associated with disorder or randomness. The Shannon entropy is nowadays considered as the formal way to define entropy. By looking to the degree sequence of a graph, one can define the entropy of a graph as well. We will talk about the graphs attaining the minimum or maximum entropy under certain restrictions and give some intuition. In particular, we sketch a proof for a conjecture by Yan.

This is joint work with Matteo Mazzamurro and Yanni Dong.

Title: Short transformations between list colourings

Presenter: Wouter Cames van Batenburg
Affiliation: TU Delft
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Often we are interested in properly colouring the vertices of a graph G using few (say at most k) colours. Typically the solution is far from unique. A natural follow-up question is how 'similar' or 'close together' these k-colourings are. This question can be formalized by studying the *reconfiguration graph* $\mathcal{C}_k(G)$, which has a vertex for each proper k-colouring of G, and two vertices of $\mathcal{C}_k(G)$ are adjacent precisely when their k-colourings differ on a single vertex of G. Much work has focused on bounding the maximum value of the diameter $\text{diam}\mathcal{C}_k(G)$ in terms of the number of vertices n of G. Cereceda (2007) famously conjectured that $\text{diam}\,\mathcal{C}_{d+2}(G) = O(n^2)$ for every d-degenerate n-vertex graph G. We consider the analogous local problem for list-colourings. We conjecture that if L is a list-assignment for a graph G with $|L(v)| \geq d(v) + 2$ for all $v \in V(G)$, then $\text{diam}\,\mathcal{C}_L(G) \leq n(G) + \mu(G)$, where $\mu(G)$ denotes the matching number of G. If true this is best possible, and thus provides an optimal strengthening of Cereceda's conjecture when restricted to regular graphs. Our first main result proves the upper bound $\text{diam}\,\mathcal{C}_L(G) \leq n(G) + 2\mu(G)$. Our second main result proves that the conjectured bound holds, whenever all v satisfy $|L(v)| \geq 2d(v) + 1$. Furthermore, we prove the conjecture for various graph classes.

Joint work with Stijn Cambie and Daniel Cranston.

Title: Random colourings of trees with constant down degree

Presenter: David de Boer
Affiliation: Kortweg-de Vries Institute, UvA
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Denote (T_d^n, r) for the rooted tree with root r, where each vertex has d children and the leaves are at distance exactly n from r. Fix a colouring with q colours of the leaves, and look at a random proper colouring of T_d^n that agrees with the colours we fixed on the leaves. What is the probability that the root gets colour 1 under such a proper colouring? Is it $1/q$, or does it depend on the fixed colouring of the leaves? If we let the distance between the root and the leaves grow, will this tend to $1/q$? After looking at this as a warm up, we will allow non proper colourings and discuss results in this setting.

This is joint work with Ferenc Bencs, Pjotr Buys and Guus Regts.

Title: Higgledy-piggledy sets in projective spaces

Presenter: Lins Denaux
Affiliation: Ghent University
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In this talk, we focus on *higgledy-piggledy sets* of k-subspaces in $\mathrm{PG}(N, q)$, i.e. sets of projective subspaces that are 'well-spread-out'. More precisely, the set of intersection points of these k-subspaces with any $(N-k)$-subspace κ of $\mathrm{PG}(N, q)$ spans κ itself. In other words, the set of points in the union of these k-subspaces forms a *strong blocking set* w.r.t. $(N-k)$-subspaces. Naturally, one would like to find a higgledy-piggledy set consisting of a small number of k-subspaces.

Although these combinatorial sets of subspaces are sporadically mentioned in older works, only since 2014 researchers have started to investigate these sets as a main point of interest. This talk aims to give its audience an overview of known results concerning higgledy-piggledy sets (lower bounds, existence results, construction methods...) and their applications to coding and graph theory, as well as share some new results and interesting open problems.

Title: The minimum and maximum graphical function-index tree problems

Presenter: Yanni Dong
Affiliation: University of Twente
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We consider the problem of finding a spanning tree that minimizes or maximizes the graphical function-index. There is a general problem: is the decision problem of the minimum or maximum graphical function-index spanning tree $\mathcal{NP}$-complete. We prove that the decision problems of the minimum and maximum spanning trees of a series of graphical function-indices are $\mathcal{NP}$-complete by a unified method.

This is joint work with Yuhang Bai, Hajo Broersma and Shenggui Zhang.

Title: Chromatic zeros of series-parallel graphs

Presenter: Jeroen Huijben
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In 2004, Sokal showed density of chromatic zeros outside the disk $|q-1| \leq 1$, by considering a particular family of series-parallel graphs. We set out to clarify the situation inside this disk for all series-parallel graphs. We managed to show density of chromatic zeros in the half plane $\Re(q) > 3/2$ and we show there exists an open region U containing the interval $(0, 32/27)$ such that $U \setminus \{1\}$ does not contain chromatic zeros.

We also disprove a conjecture of Sokal by showing that for each large enough integer Δ there exists a series-parallel graph for which all vertices but one have degree at most Δ and whose chromatic polynomial has a zero with real part exceeding Δ.

This is joint work with Ferenc Bencs and Guus Regts (arXiv:2204.10038)

Title: Enumerating Coprime Polynomials over $GF(2)$ with Nonzero Constant Term

Presenter: Luca Mariot
Affiliation: Digital Security Group, Radboud University
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The construction of relatively prime polynomials over finite fields has several applications in cryptography, coding theory and combinatorics. In the binary field $GF(2)$, Benjamin and Bennett showed a simple bijection between coprime/non-coprime polynomial pairs based on what they call DilcuE's algorithm (i.e., Euclid's algorithm ran backwards). This allows one to easily construct a pair of coprime polynomials starting from a non-coprime one by executing Euclid's algorithm as usual, change the last remainder to 1, and then go backward with DilcuE's algorithm by applying the same sequence of quotients.

In this talk, we consider a specific case of the above problem, namely the enumeration of coprime polynomials over $GF(2)$ where both polynomials have the same degree and a nonzero constant term, motivated by the construction of orthogonal Latin squares through linear cellular automata (CA). We decompose our problem of interest in two parts. The first part consists in enumerating the sequences of constant terms for the quotients visited by DilcuE's algorithm. We show that such sequences form a regular language recognized by a finite state automaton, whose transitions are described by a De Bruijn graph. Then, we employ classic results from algebraic language theory to determine the generating function of this regular language, and derive the corresponding closed-form recurrence to count the number of all sequences of fixed length. The second part concerns the enumeration of the remaining coefficients in the quotients generated by DilcuE's algorithm. We show that this is equivalent to enumerate all compositions of the polynomials' degree. Putting everything together, we finally devise a combinatorial algorithm that enumerates all pairs of coprime polynomials with nonzero constant term, by independently generating all sequences of constant terms and compositions.

This is joint work with Enrico Formenti (Université Côte d'Azur).

Title: New lower bounds on crossing numbers of $K_{m,n}$ from permutation modules and semidefinite programming

Presenter: Sven Polak
Affiliation: Centrum Wiskunde & Informatica
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Abstract: In this talk we explain how to use semidefinite programming and representation theory to compute new lower bounds on the crossing number of the complete bipartite graph $K_{m,n}$, extending a method from de Klerk et al. [SIAM J. Discrete Math. 20 (2006), 189–202] and extending the subsequent reduction by De Klerk, Pasechnik and Schrijver [Math. Prog. Ser. A and B, 109 (2007) 613–624].

We exploit the full symmetry of the problem by developing a block-diagonalization of the underlying matrix algebra and use it to improve bounds on several concrete instances. Our results imply that $\mathrm{cr}(K_{10,n}) \geq 4.87057n^2 - 10n$, $\mathrm{cr}(K_{11,n}) \geq 5.99939n^2 - 12.5n$, $\mathrm{cr}(K_{12,n}) \geq 7.25579n^2 - 15n$, $\mathrm{cr}(K_{13,n}) \geq 8.65675n^2 - 18n$ for all n. The latter three bounds are computed using a relaxation of the original semidefinite programming bound, by only requiring one small matrix block to be positive semidefinite. Our lower bound on $K_{13,n}$ implies that for each fixed $m \geq 13$, $\lim_{n\to\infty} \mathrm{cr}(K_{m,n})/Z(m,n) \geq 0.8878m/(m-1)$. Here $Z(m,n)$ denotes the *Zarankiewicz* number: the conjectured crossing number of $K_{m,n}$.

This is joint work with Daniel Brosch (Tilburg University).

Title: Cycles in Mallows random permutations

Presenter: Teun Verstraaten
Affiliation: Bernoulli Institute, Groningen University, The Netherlands
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We study cycle counts in permutations of $1, \ldots, n$ drawn at random according to the Mallows distribution. Under this distribution, each permutation $\pi \in S_n$ is selected with probability proportional to $q^{\text{inv}(\pi)}$, where $q > 0$ is a parameter and $\text{inv}(\pi)$ denotes the number of $i < j$ such that $\pi(i) > \pi(j)$. For ℓ fixed, we study the vector $(C_1(\Pi_n), \ldots, C_\ell(\Pi_n))$ where $C_i(\pi)$ denotes the number of cycles of length i in π and Π_n is sampled according to the Mallows distribution.

Here we show that if $0 < q < 1$ is fixed and $n \to \infty$ then there are positive constants m_i such that each $C_i(\Pi_n)$ has mean $(1 + o(1)) \cdot m_i \cdot n$ and the vector of cycle counts can be suitably rescaled to tend to a joint Gaussian distribution. Our results also show that when $q > 1$ there is a striking difference between the behaviour of the even and the odd cycles. The even cycle counts still have linear means, and when properly rescaled tend to a multivariate Gaussian distribution. For the odd cycle counts on the other hand, the limiting behaviour depends on the parity of n when $q > 1$. Both $(C_1(\Pi_{2n}), C_3(\Pi_{2n}), \ldots)$ and $(C_1(\Pi_{2n+1}), C_3(\Pi_{2n+1}), \ldots)$ have discrete limiting distributions – they do not need to be renormalized – but the two limiting distributions are distinct for all $q > 1$. We describe these limiting distributions in terms of Gnedin and Olshanski's bi-infinite extension of the Mallows model.

This is joint work with Tobias Müller.

Title: On a nonabelian Kneser theorem

Presenter: Maximilian Wötzel
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The well known theorem of Kneser, stating that small sumsets in abelian groups must be periodic, does not hold in nonabelian groups. We prove that in a finite nonabelian group G, a weaker version of Kneser's theorem does hold, stating that if a symmetric set $1 \in S = S^{-1} \subset G$ satisfies $|S^2| < 2|S| - 1$ then S^2 is almost periodic. This is shown in the more general context of vertex transitive graphs. Perhaps surprisingly, the corresponding statement for infinite vertex transitive graphs turns out to be false.

This is joint work with Oriol Serra.

Title: Anti-Ramsey numbers for vertex-disjoint triangles

Presenter: Fangfang Wu
Affiliation: University of Twente
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An edge-colored graph is called rainbow if all the colors on its edges are distinct. Given a positive integer n and a graph G, the anti-Ramsey number $ar(n, G)$ is the maximum number of colors in an edge-coloring of K_n with no rainbow copy of G. Denote by kC_3 the union of k vertex-disjoint copies of C_3. In this talk, we determine the anti-Ramsey number $ar(n, kC_3)$ for $n = 3k$ and $n \geq 2k^2 - k + 2$, respectively. When $3k \leq n \leq 2k^2 - k + 2$, we give lower and upper bounds for $ar(n, kC_3)$

This is joint work with Shenggui Zhang, Binlong Li and Jimeng Xiao.